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~~141.~~ On the asymptotic property of the transformed functions

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Titchmarsh: theory of Fourier integral P. 172 ニ Fourier tranformed 函数 $Fc(x)$ 及ビ $Fs(x)$ ノ漸近値ノ定理デ次ノヨウナモノガアル.

定理　$f(x)=x^{-\alpha}\phi(x)$　　　$(0<\alpha<1)$

ココニ $\phi(x)$ ハ bounded variation ナ函数 $(0, \infty)$ ナラバ

$$Fc(x)\sim\phi(+0)\sqrt{\left(\frac{2}{\pi}\right)}\Gamma(1-\alpha)\sin\frac{1}{2}\pi\alpha\, x^{\alpha-1}\quad x\to\infty$$

$$Fc(x)\sim\phi(\infty)\sqrt{\left(\frac{2}{\pi}\right)}\Gamma(1-\alpha)\sin\frac{1}{2}\pi\alpha\, x^{\alpha-1},\quad x\to 0$$

又 $Fs(x)$ ハ同ジ條件式ニテ. $\sin\frac{1}{2}\pi\alpha$ ヲ $\cos\frac{1}{2}\pi\alpha$ ニ換ユレバ良イ.

茲ニ論ジタイノハ更ニ一般的ナ Transformed function ニモ拡張出来. 特別ナ場合トシテ Fourier tran. Hankel-Bessel transformation 及ビ Laplace transformation ノ asymtotic value ヲ含ム.

一般ナ定理

$$g(x)=\int_0^\infty f(y)\, k(xy)\, dy.$$

ココニ $k(x)$ ハ kernel デアリ. 次ノ條件ヲ満足スル

$$\int_0^\infty y^\beta k(y)\, dy \text{ ハ } \Psi(\beta) \text{ ニ收斂シ.}$$

$f(y)=y^\beta\varphi(y)$ ニテ $\varphi(y)$ ハ non-increasing function $(0, \infty)$ トス.

ソノトキ $g(x) \sim \varphi(+0)\,\psi(\beta)\,x^{-\beta-1}$ as $x \to \infty$

証明ハ殆ド Titchmarsh ト同ジ (second mean value 定理ヲ使フ)

証明 $g(x) = \int_0^\infty f(y)\,k(xy)\,dy$

$$g(x) = \int_0^\infty y^\beta \varphi(y)\,k(xy)\,dy$$

$$= \int_0^\infty \left(\frac{y}{x}\right)^o \varphi\left(\frac{y}{x}\right) k(y)\,\frac{dy}{x}$$

$$= x^{-s-1} \int_0^\infty \varphi\left(\frac{y}{x}\right) y^\beta k(y)\,dy\,.$$

$$\int_0^\infty \varphi\left(\frac{y}{x}\right) y^\beta k(y)\,dy = \left(\int_0^\Delta + \int_\Delta^\infty\right) \varphi\left(\frac{y}{x}\right) y^\beta k(y)\,dy = I_1 + I_2$$

$$I_2 = \int_\Delta^\infty \varphi\left(\frac{y}{x}\right) y^s k(y)\,dy$$

$$= \varphi\left(\frac{\Delta}{x}\right) \int_\Delta^{\Delta'} y^s k(y)\,dy \to 0 \quad \text{as } \Delta \to \infty$$

$$= o(\Delta)$$

$$\int_0^\Delta \left[\varphi(+0) - \varphi\left(\frac{y}{x}\right)\right] y^\beta k(y)\,dy$$

$$= \left\{\varphi(+0) - \varphi\left(\frac{\Delta}{x}\right)\right] \int_\delta^\Delta y^\beta k(y)\,dy \to 0 \quad \text{as } x \to \infty$$

$$I_1 \to \varphi(+0) \int_0^\infty y^\beta k(y)\,dy \quad \text{as } \Delta \to \infty$$

$$g(x) \sim \varphi(+0)\ \psi(\beta)\, x^{-\beta-1}$$

Titchmarsh ノ定理ハ.

$$f(x) = \sqrt{\frac{2}{\pi}}\ x^{-\alpha}\ \varphi(x)$$

$$k(x) = \cos x \qquad \psi(\beta) = \sqrt{\frac{2}{\pi}} \int_1^\infty y^{-\alpha} \cos y\, dy$$

$$\psi(\beta) = \Gamma(1-\alpha)\sin\tfrac{1}{2}\pi\alpha \qquad \beta = -\alpha$$

$$g(x) \sim \phi(+0)\sqrt{\left(\tfrac{2}{\pi}\right)}\;\Gamma(1-\alpha)\sin\tfrac{1}{2}\pi\alpha \cdot x^{\alpha-1}$$

as $x \to \infty$

Example トシテ

1 Laplace transformation

$$g(x) = \int_0^\infty f(y)\, e^{-xy}\, dy$$

$$k(x) = e^{-x}$$

$$f(x) = x^\beta\, \varphi(x)$$ ト置キ.

$\varphi(x)$ ハ non-increasing $(0, \infty)$

$$g(x) \sim \varphi(+0)\left(\int_0^\infty y^\beta e^{-y}\, dy\right) x^{-1-\beta}$$

$$\sim \varphi(+0)\,\Gamma(1+\beta)\, x^{-1-\beta} \qquad \text{as } x \to \infty$$

2. Hankel-Bessel transformation

$k(x) = x^{\frac{1}{2}} J_\nu(x)$ $\qquad$ $J_\nu(x)$ ハ Bessel function.

$$f(y) = \varphi(y)\; y^{\alpha-\nu-1-\frac{1}{2}}$$

$$g(x) = x^{-1+\nu+1+\frac{1}{2}-\alpha} \int_0^\infty \varphi\left(\frac{y}{x}\right) y^{\alpha-\nu-1} J_\nu(y)\, dy$$

$$g(x) \sim \varphi(+0) \int_0^\infty y^{\alpha-\nu-1} J_\nu(y)\, dy \cdot x^{\nu+\frac{1}{2}-\alpha}$$

$$\sim \varphi(+0)\, \frac{2^{a-\nu-1}\, \Gamma(\frac{1}{2}a)}{\Gamma(\nu-\frac{1}{2}a+1)} \cdot x^{\nu+\frac{1}{2}-\alpha}$$

as $x \to \infty$

3. Struves kernel $\quad k(x) = x^{\frac{1}{2}} H_\nu(x)$,

$$H_\nu(x) = \sum_{n=0}^\infty \frac{(-1)^n \left(\frac{1}{2}x\right)^{\nu+2n+1}}{\Gamma(n+\frac{3}{2})\,\Gamma(\nu+n+\frac{3}{2})} \qquad \nu > -\frac{3}{2}$$

$$g(x) = \int_0^\infty f(y)\sqrt{(xy)}\; H_\nu(xy)\, dy$$

$$f(y) = \varphi(y) \quad y^{\alpha-\nu-1-\frac{1}{2}}$$

$\varphi(y)$ is non-increasing function.

$$g(x) = x^{-1}\int_0^\infty f\left(\frac{y}{x}\right) k(y)\,dy$$

$$= x^{\nu+\frac{1}{2}-\alpha}\int_0^\infty \varphi\left(\frac{y}{x}\right) y^{\alpha-\nu-1}\ H_\nu(y)\,dy$$

$$g(x) \sim \varphi(+0)\left(\int_0^\infty y^{\alpha-\nu-1}\ H_\nu(y)\,dy\right) x^{\nu+\frac{1}{2}-\alpha}$$

$$\sim \varphi(+0)\ \frac{2^{\alpha-\nu-1}\,\Gamma(\frac{1}{2}\alpha)\tan\frac{1}{2}\alpha\pi}{\Gamma(\nu-\frac{1}{2}+1)}\, x^{\nu+\frac{1}{2}-\alpha}$$

$$\left(-1 < \alpha < \nu + \frac{3}{2}\right)$$

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Reference Book Titchmarsh. Theory of Fourier integral. P 172. 182. 212